Lesson Two: "*Converting Numbers into Exponential Products*"

Integer Exponents: Positive and Negative Powers

Positive Exponents

An exponent tells us how many times to multiply the base number.

$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

In this example, our base is 3, and our exponent is 5. This tells us to multiply the number three, by itself, five times.

REMEMBER: Exponents only expand what they directly touch!

Note: Multiplication is repeated addition, so: $\mathbf{2 + 2 + 2 + 2 = 2 \cdot 4 = 8}$

Exponents are repeated *multiplication*, so: $\mathbf{2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16}$

Negative Exponents

A negative exponent is not what it seems…

$2^{-3} = ?$

It is not possible to expand our base of *2*, negative *3* times as we would with positive exponents. Negative exponents mean we have an extra job to do before we can expand them.

$$2^{-3} = \frac{1}{2^3} = ?$$

The negative sign tell us that the number (with its exponent) is in the wrong place…**They need to move.** Consider a number with a negative exponent as being unhappy where he is. He who has the negative exponent is in the wrong/opposite location. To fix this, imagine there is a top and bottom (upstairs and downstairs), and once you move the number to its opposite location it will be happy.

$$2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$$

Important Vocabulary

Prime Number:

A **prime number** can be *divided* evenly only by 1 or itself.
It must be a whole number greater than 1.

The first few *prime numbers* are: 2, 3, 5, 7, 11, 13, and 17.

Factors:

Factors are the numbers you **multiply** together to get *another number*

For example:

FACTOR

$$2 \cdot 3 = 6$$

FACTOR

Therefore, the factors of 6 are 2 and 3.

Prime Factorization:

"Prime Factorization" is finding **which prime numbers** *multiply* together to make the *original number*.

For example:

What are the factors of 12?

$$12 = 4 \cdot 3$$

It is true that $4 \cdot 3 = 12$. However, 4 is not prime, it can be broken down some more…

So…

$$12 = 4 \cdot 3 = 2 \cdot 2 \cdot 3$$

Therefore the *factors* of **12** are 2, 2, and 3.

Prime Factorization & Exponents: Using Prime Factorization to Rewrite in Exponential Form

Problem 1: What is the prime factorization of **147** ?

$147 = 7 \cdot 21 = 7 \cdot 7 \cdot 3$

Therefore, the factors of 147 are **7**, **7** and **3**. These prime factors are unique to the number *147*.

Step 1: Create a factor tree for the given number

Step 2: Group the common factors

Step 3: Rewrite using exponent notation

Step 4: Check solution

Problem 2: Write 144 as a product of its prime factors.

$144 = 12 \cdot 12 = 3 \cdot 4 \cdot 3 \cdot 4 = 3 \cdot 2 \cdot 2 \cdot 3 \cdot 2 \cdot 2$

The factors of *144* are *3, 3, 2, 2, 2,* and *2.*

Now we need to **express 144** as a product of its prime factors. This where our *properties of exponents* come into play.

$3 \cdot 3 = 3^2$ and $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

SOLUTION: $3^2 \cdot 2^4$

We can **check** our solution by multiplying our numbers together. Does $3^2 \cdot 2^4 = 144$?

$3^2 \cdot 2^4 = 9 \cdot 16 = 144$ ☺ Yes!!

Problem 3: Write 1/81 as a product of its prime factors.

$$\frac{1}{81} = \frac{1}{9 \cdot 9} = \frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{3^4} = 3^{-4}$$

Remember our special friend the negative exponent!

Problem 4: Write 9/625 as a product of its prime factors.

$$\frac{9}{625} = \frac{3 \cdot 3}{125 \cdot 5} = \frac{3 \cdot 3}{25 \cdot 5 \cdot 5} = \frac{3 \cdot 3}{5 \cdot 5 \cdot 5 \cdot 5} = \frac{3^2}{5^4} = 3^2 \cdot 5^{-4}$$